

## Multimodal Transfer Matrix Approach for the Analysis of Glide Symmetric Dielectric/Magnetic Structures

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### Summary

In this work, the Multimodal Transfer Matrix Approach (MMTMA) is used to investigate two-dimensional dielectric and magnetic periodic structures that possess glide symmetries. This method is able to retrieve both the phase shift and attenuation constant of any arbitrary structure and, moreover, it enables a physical insight of the operation of periodic structures. Here, we propose the MMTMA to explain the differential response produced by dielectric/magnetic glide symmetries, showing some interesting properties for the design of 2-D lenses produced with additive manufacturing.

## 1 Introduction

In recent years, periodic structures that exhibit higher-order symmetries have attracted attention for their possibility to introduce beneficial modifications in their electromagnetic properties. A periodic structure possesses glide symmetry if it is invariant after translation and mirroring [1]. The remarkable properties of glide-symmetric periodic structures have been demonstrated, including widened stopbands, reduced dispersion, as well as enhanced anisotropy and magnetic response [2–5]. Here, we propose and study a new class of glide-symmetric periodic structures which are made of a 2-D composite dielectric/material embedded in a parallel plate waveguide (PPW) configuration. MMTMA [6] is used to explain the operation of the corresponding unit cell.

## 2 Multimodal Transfer Matrix Approach (MMTMA)

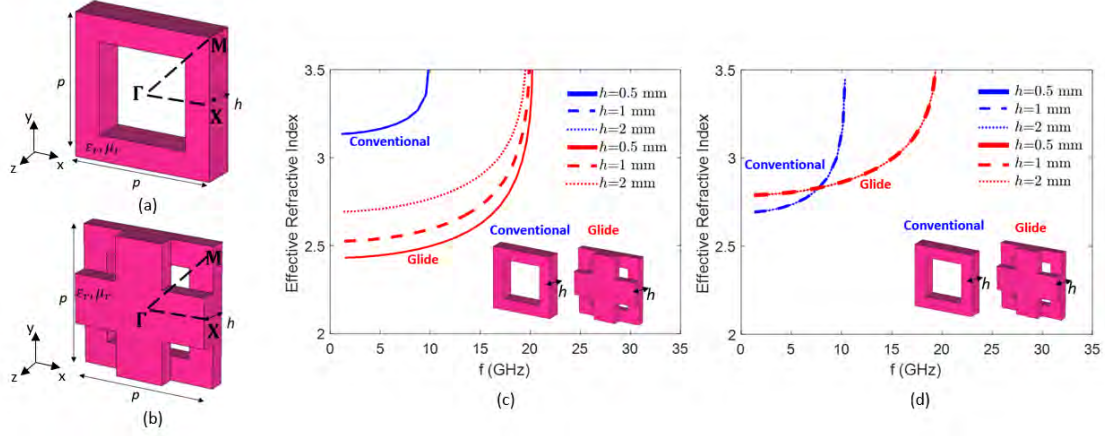
MMTMA is a hybrid method that processes the generalized scattering matrix obtained with commercial simulation software to compute the dispersion diagrams of periodic structures. In 2-D unit cells of a periodic structure, Floquet boundary conditions are imposed on the two periodicity axes for voltages and currents on the input/output ports. We assume that the unit cell is periodic in the  $xy$ -plane, as shown in Fig.1, and it is square with period  $p$  along both axes. Following the linearization procedure proposed in [7], it is possible to transform the original non-linear eigenvalue problem associated with 2-D structures to a standard one that can easily be solved without the need of a zero-searching algorithm. Assuming  $\gamma_y = 0$  (condition satisfied along the  $\Gamma\bar{X}$  segment), the  $2N$ -rank linear problem has the following expression ( $N$  denotes the number of input/output port modes):

$$[\tilde{\mathbf{T}}_{xx} - \tilde{\mathbf{T}}_{xy}(\tilde{\mathbf{T}}_{yy} - \mathbf{U})^{-1}] \mathbf{F}_x = \lambda_x \mathbf{F}_x \quad (1)$$

where  $\lambda_x = e^{\gamma_x p}$ , and  $[\tilde{\mathbf{T}}] = \begin{bmatrix} \tilde{T}_{xx} & \tilde{T}_{xy} \\ \tilde{T}_{yx} & \tilde{T}_{yy} \end{bmatrix} = [\tilde{\mathbf{P}}][\mathbf{T}][\tilde{\mathbf{P}}]^T$ , with  $[\tilde{\mathbf{P}}]$  being a permutation matrix and  $\mathbf{F}_x = [\mathbf{V}_x, \mathbf{I}_x]^T$ . The problem associated with the  $\Gamma\bar{M}$  segment is already linear since  $\gamma_y = \gamma_x$ .

## 3 Numerical results

The new glide-symmetric unit cell depicted in Fig. 1(b) is analyzed. It is made up of two pieces of dielectric/magnetic material embedded in a parallel plate waveguide. The unit cell is 2-D glide-symmetric in the  $x$  and  $y$  directions, with the glide plane located in  $z = h/2$ . It is an extension of the one reported in [8], which now includes the possibility of having magnetic materials. Fig. 1(a) shows the structure which does not have glide symmetry and is denoted as *conventional*. The dielectric and/or magnetic slabs of the unit cells are perforated with square air cavities. The size of these cavities is defined by the parameter  $w$ , which in this paper is  $w = 1.5$  mm. The period of the square cell is  $p = 4.2$  mm. Figs. 1(c) and (d) show the calculated effective refractive index versus the frequency for conventional and glide-symmetric structures made of only dielectric or magnetic materials. This refractive index was calculated with the MMTMA. Several thickness values  $h$  are considered. It is possible to observe that, for the conventional cases, the effective refractive index does not vary as the thickness  $h$  of the structure changes in both the dielectric and magnetic unit cells. Instead, in the case of glide-symmetric dielectric materials, the height of the structure has a



**Figure 1.** Dielectric/magnetic unit cells of (a) conventional and (b) glide-symmetric periodic structures. Effective refractive index for the conventional (blue lines) and glide (red lines) magnetic structures, for several values of the thickness  $h$ ;  $w = 1.5$  mm,  $p = 4.2$  mm. (c) Dielectric structure:  $\epsilon_r = 16$ ,  $\mu_r = 1$ . (d) Magnetic structure:  $\epsilon_r = 1$ ,  $\mu_r = 16$ .

strong impact on the dispersion properties. These results can be understood with the help of MMTMA, which allows us to know the different waveguide modes that are excited in the 2-D parallel plate configurations. Indeed, the impact of the height of the dielectric structure on the dispersion properties is due to the interaction of the needed modes with the dielectric properties of the structure. This does not apply to the case of the conventional periodic structure considered in this work since it does not have vertical variations. Also, when the unit cell is made of only magnetic materials, since the fundamental modes are not associated with magnetic conditions, the height has no impact on the dispersion properties. This differential response is interesting for practical applications, such as the design of 2-D lens antennas.

## 4 Acknowledgements

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